Upstream pricing strategies, multiple inputs, and downstream delegation*

Estrategias de precios aguas arriba, insumos múltiples y delegación aguas abajo

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Abstract

This paper considers a delegation game between one multi-input firm and one single-input firm engaging in Cournot competition in the downstream market. Both firms purchase a standard input from a core input supplier, and the multi-input firm also needs a supplementary input provided by an independent supplier. I study two input pricing policies of the core input supplier, uniform pricing and third-degree price discrimination, and obtain the following. First, regardless of the upstream pricing strategies, both downstream firms delegate in equilibrium, but contrary to traditional analysis, delegation is mutually profitable. Second, the core input supplier prefers uniform pricing to third-degree price discrimination. Lastly, uniform pricing is more socially desirable than discriminatory pricing.

Key words: *Downstream delegation; multiple inputs; uniform vs discriminatory input pricing.*

JEL Classification: *L13, L21, M11.*

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Resumen

Este documento considera un juego de delegación entre una empresa de múltiples insumos y una empresa de un solo insumo que participa en competencia a la Cournot en el mercado aguas abajo. Ambas empresas compran un insumo estándar de un proveedor principal y la empresa de insumos múltiples también necesita un insumo suplementario proporcionado por un proveedor independiente. Estudio dos políticas de precios de insumos del proveedor principal de insumos: precios uniformes y discriminación de precios de tercer grado. Los resultados encontrados son: En primer lugar, independientemente de las estrategias de fijación de precios aguas arriba, ambas empresas aguas abajo delegan en equilibrio, pero contrariamente al análisis tradicional, la delegación es mutuamente beneficiosa. En segundo lugar, el proveedor principal de insumos prefiere la fijación uniforme de precios a la discriminación de precios en tercer grado. Por último, la fijación uniforme de precios es más deseable desde el punto de vista social que la fijación de precios discriminatorios.

Palabras clave: *Delegación aguas abajo; insumos múltiples; precios de insumos uniformes vs discriminatorios.*

Clasificación JEL: *L13, L21, M11.*

1. Introduction

In modern economies, the separation of ownership and control can be frequently observed in big companies (Coffee, 2001). Seminal papers by Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987), hereafter VFJS, support this evidence by showing that both firms in a duopoly model choose to delegate the output/price decision to managers in equilibrium. Moreover, VFJS also prove that in a delegation game with a Cournot duopoly, the delegation contracts incentivize the managers to be more aggressive than profit-maximizers such that both firms become worse off than without delegation. These findings are widely accepted in the literature on managerial delegation (Lambertini, 2017).

The results of VFJS are challenged by studies incorporating vertically related markets, e.g., Park (2002) and Liao (2010).[1] Both papers build a model with an upstream monopolist and two downstream firms and allow reve-

[1] In the absence of the upstream market, Colombo (2019, 2022) also proves that the result of VFJS may not hold in a Cournot duopoly with cost and quality asymmetry, respectively. Specifically, he shows that the more efficient (high-quality) firm is more profitable post-delegation if the cost (quality) differential is sufficiently high.

nue delegation in the downstream market by following Fershtman and Judd (1987) and Sklivas (1987).[2] Assuming the downstream firms' marginal costs only contain the uniform input price charged by the upstream monopolist, Park (2002) shows that no firm delegates in equilibrium. Liao (2010) then extends the analysis to the case where each downstream firm bears a marginal transformation cost in addition to the input price and accepts both uniform pricing and third-degree price discrimination in the upstream market. The author demonstrates that delegation occurs in equilibrium and is mutually unprofitable (profitable) for Cournot firms under uniform input pricing (upstream price discrimination).

It should be noted that both Park (2002) and Liao (2010) only discuss single-input downstream firms, which are symmetric. However, it is more frequently observed that firms are asymmetric in the real world, and the asymmetry may come from technologies where the number of inputs used to generate the final goods is different.[3] I, therefore, contribute to this strand of literature by constructing a downstream delegation framework involving a multi-input downstream manufacturer competing against a single-input rival in a Cournot competition. Two upstream monopolists provide a core and a supplementary input to the multi-input downstream firm, while the single-input manufacturer's production requires the core input only.[4] The core input supplier can either charge uniform prices or third-degree price discrimination. Unlike Park (2002) and Liao (2010), I follow Vickers (1985) to take into account output delegation in the downstream market.[5]

Three main findings are obtained from the analysis. First, delegation with a less aggressive incentive parameter emerges as a dominant strategy, and it

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2 Revenue delegation is a type of delegation contract where the manager's performance is measured by a combination of the firm's profit and revenue.

3 Using more inputs may help improve product quality. However, production by outdated technologies may sometimes employ more inputs to correct disadvantages such as labor, security, IT support, and maintenance services. Please refer to the website: https://www.coxblue.com/how-outdated-technology-costs-businesses-more-than-it-saves/

4 There is a strand of literature concerning complementary inputs (e.g., Kopel *et al*., 2016, 2017; Kitamura *et al*., 2018; Lin *et al*., 2022). Kopel *et al*. (2016, 2017) consider the sourcing strategy of a multi-input-multi-product firm. Kitamura *et al*. (2018) construct a model with a complementary input monopolist to discuss the existence of an exclusive contract in the core input segment. Lin *et al*. (2022) investigate the vertical licensing behavior of a multi-input firm and its welfare effects. To the best of my knowledge, research has yet to incorporate complementary input into the delegation game.

5 Output delegation is a type of delegation contract where the manager's performance is measured by a combination of the firm's profit and output quantity. Because it is hard to provide clear-cut results by adopting revenue delegation, and this paper focuses on discovering the change in competition intensity post-delegation only, I employ output delegation instead.

turns out to be mutually profitable for the downstream firms, regardless of input pricing strategies. Second, uniform pricing is superior to third-degree price discrimination from the viewpoint of the core input supplier. Lastly, uniform input pricing is more socially desirable than price discrimination.

Similar to Liao (2010), this study finds that delegation is mutually profitable under third-degree input price discrimination. This finding is significantly based on the *vertical externality effect*. This effect suggests that the input supplier will charge a higher (lower) price for the firm competing more (less) aggressively in the downstream market. Hence, the downstream firms can set less aggressive incentive parameters in the delegation contracts. In doing so, they benefit from lower input prices and the fall in competition intensity in the downstream market. As a result, the downstream firms are better off post-delegation.[6] The presence of a supplementary input may influence the multi-input firm to compete less aggressively to lower the price offered by the supplementary input supplier. In the meantime, the single-input firm is also incentivized to reduce its combativeness to increase the input price from the supplementary input supplier, leading to a higher marginal cost for its rival. So, this paper creates an extra effect, namely the *supplementary input effect*, which has the same impact as the vertical externality effect and strengthens Liao's (2010) result under third-degree input price discrimination.

However, Liao's (2010) and VFJS's results do not hold in the current model, given the core input supplier charges a uniform input price, as delegation becomes mutually profitable. The intuition can be explained as follows. As the downstream firms pay the same price under uniform input pricing, a downstream firm, in an effort to lower the input price by reducing its aggressiveness, will also lessen its rival's marginal cost, thereby indirectly harming itself due to strategic substitutes. Thus, each firm will compete more severely to strategically weaken the opposition. This is the so-called *spillover effect*.[7] This effect induces the optimal incentive parameter in Park (2002) to be profit-maximizing when the downstream production incurs no marginal costs except input prices. By considering an identical extra marginal transformation cost for both downstream firms, Liao (2010) shows that the spillover effect helps outweigh the vertical externality effect such that the downstream managers compete more aggressively post-delegation. The overturned result can occur by introducing

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6 Choi *et al*. (2020) also derive the reverse result to VFJS´s based on the vertical externality effect. They encompass two vertical chains into a model in which only the downstream firms can select optimal delegation strategies. Delegation arises in equilibrium with less aggressive managers under linear input pricing. It, therefore, turns out to be mutually profitable.

7 The spillover effect under uniform input pricing is well established in Bernhofen (1997).

the supplementary input effect, which influences the managers to behave less forcefully, making delegation mutually profitable.

Although this paper mainly contributes to the literature on managerial delegation, it also provides two valuable results in comparing uniform input pricing and third-degree input price discrimination. First, from the viewpoint of the core input supplier, this research derives a different result from the conventional wisdom, which suggests that a monopolistic seller prefers third-degree price discrimination to uniform pricing because of the more instruments to extract rents from buyers in the former strategy (Holmes, 1989).[8] In addition, Brito *et al*. (2019) show that input price discrimination benefits the upstream monopolist if downstream quality asymmetry exists. The reverse result can occur by replacing quality asymmetry with technological asymmetry. Second, from the viewpoint of social planners, this research supports the outcome of DeGraba (1990) and Valletti (2003) by showing that uniform input pricing results in a higher welfare level than third-degree input price discrimination, while other papers can unearth some circumstances for the opposite finding (e.g., Arya and Mittendorf, 2010; Kao and Peng, 2012; Brito *et al*., 2019; and Miklós-Thal and Shaffer, 2021).

The remainder of this paper is structured as follows. Section 2 describes the model with downstream homogeneous products. Section 3 makes a comparison between the two pricing regimes. A discussion on vertically differentiated products in the downstream market is provided in Section 4. Finally, Section 5 concludes the paper.

2. The Model Of Downstream Homogeneous Products

Two downstream manufacturers, firms 1 and 2, produce substitute products as goods 1 and 2, respectively. Following Singh and Vives (1984), a representative consumer is assumed to enjoy a utility function from buying the products as follows:

$$U = a_1 q_1 + a_2 q_2 - \frac{1}{2}(q_1 + q_2)^2 + z \tag{1}$$

where q_i (i = 1, 2) denotes the output quantity of good i, a_i represents consumers' willingness to pay, which could be referred to as the quality level of good i (Häckner, 2000), and z stands for the numeraire.

The inverse demand system for the final goods is derivable as:

8 Theoretical analyses involving vertically related markets, such as Valletti (2003) and Liao (2010), also prove that an upstream monopolist will be more profitable by committing to uniform pricing if the downstream firms are symmetric.

(2) $$p_i = a_i - q_i - q_j ; i, j = 1,2, i \neq j$$

where p_i is the price of good *i*. For simplicity, assume that $a_1 = a_2 = 1$ such that the products are homogeneous. In Section 4, I show that the main results are robust when the downstream firms produce vertically (quality) differentiated products.

Goods 1 and 2 now face the same elastic demand function $p(Q) = 1 - Q$, where p and Q denote the price and total quantity demanded, respectively, and $Q = q_1 + q_2$. The downstream firms are asymmetric in terms of technology. To generate one unit of the downstream product, firm 1 employs one unit of a core input (input *C*) and one unit of a supplementary input (input *S*). In comparison, the production of firm 2 only requires the core input based on a one-to-one relationship. Input *C* (*S*) is provided by a monopolist, denoted as supplier *C* (*S*), with linear wholesale pricing. For simplicity, input supplier *m*'s (*m* = *C*, *S*) marginal cost is assumed to be *nil*, and manufacturer *i*'s marginal cost (*i* = 1, 2) contains the input price(s) only.

The input suppliers' profit functions are expressed as:

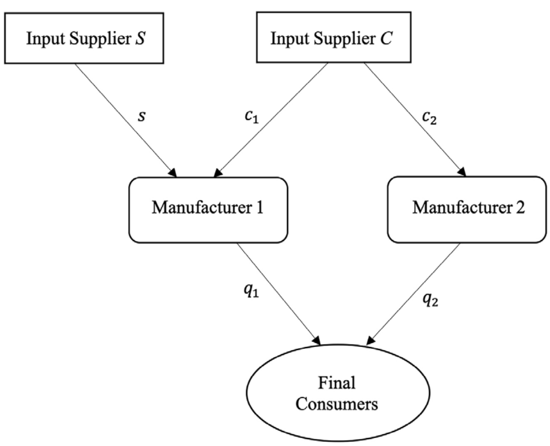
(3.1) $$\pi_C = c_1 q_1 + c_2 q_2$$

(3.2) $$\pi_S = s q_1$$

where c_i (*i* = 1, 2) is the per-unit input price that firm *i* pays to supplier *C*, and *s* denotes the per-unit input price that firm 1 pays to supplier *S*. I draw Figure 1 below to describe the trading framework of this paper.

FIGURE 1

THE TRADING FRAMEWORK



Following Vickers (1985), I assume that manufacturer *i*'s owner (*i* = 1, 2) can hire a manager and delegate the output decision to him. In doing so, the owner evaluates the manager's performance by a function that contains not only the firm´s profit but also a fraction θ_i of its output quantity.[9] The downstream firms' profit functions and the managers' objective functions are defined as follows:

(4.1) $$\pi_i = \left(p - k_i\right) q_i$$

(4.2) $$V_i = \pi_i + \theta_i q_i, i = 1,2$$

where $k_1 = c_1 + s$ and $k_2 = c_2$ are the downstream firms' marginal costs. Note that when $\theta_i > (<) 0$, firm *i*'s manager competes more (less) aggressively than a profit-maximizer in the output market, and $\theta_i = 0$ indicates the case where firm *i*'s owner chooses not to delegate, i.e., $V_i = \pi_i$.

The game in question consists of three stages. In stage 1, each downstream firm's owner determines whether to delegate the output decision to a manager or not. If the owner chooses to do so, an optimal incentive parameter will be assigned to maximize profit. Next, suppliers *C* and *S* offer input prices to the downstream firms in the second stage. Lastly, firms 1 and 2 compete in quantity in stage 3.[10] We can use the typical backward induction to solve the game.

In what follows, I analyze two different regimes depending on the pricing policy of the core input supplier. They are uniform input pricing and third-degree input price discrimination. Appendix A shows that the result under discriminatory input pricing is consistent with those in Liao (2010) and Choi *et al*. (2020), in which the downstream firms become less aggressive post-delegation, and delegation is mutually profitable. Hence, this section focuses more on the regime of uniform input pricing.

Suppose supplier *C* commits to uniformly treating the downstream producers, i.e., $c_1 = c_2 = c$. In stage 3, by differentiating V_i in (4.2) with respect to q_i (*i* = 1, 2), we obtain:

(5) $$\frac{\partial V_i}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} + \theta_i = p - k_i + \frac{\partial p}{\partial q_i} q_i + \theta_i = 0; i = 1,2$$

Solving (5) for the optimal downstream output quantities yields:

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9 Other managerial economists also adopt this type of delegation. See, for example, Matsumura and Matsushima (2012), Nakamura (2012), Jansen *et al*. (2015), and Fanti *et al*. (2017).

10 The analysis is limited to the Cournot case because only then delegation by both firms ends up being mutually harmful. The full analysis under Bertrand downstream competition is left for future research.

(6) $$\tilde{q}_1 = \frac{1-\tilde{c}-2\tilde{s}+2\tilde{\theta}_1-\tilde{\theta}_2}{3}, \tilde{q}_2 = \frac{1-\tilde{c}+\tilde{s}-\tilde{\theta}_1+2\tilde{\theta}_2}{3}$$

where the accent ~ stands for variables under the uniform input pricing regime.

We proceed to stage 2, where the upstream firms set optimal input prices. Note that under the uniform input pricing regime, supplier *C*'s profit function can be rewritten as $\tilde{\pi}_C = \tilde{c}\tilde{Q}$, while that of supplier *S* stays the same as in (3.2). By substituting (6) into the upstream firms' profit functions, then differentiating $\tilde{\pi}_C$ and $\tilde{\pi}_S$ with respect to $\tilde{c}$ and $\tilde{s}$, respectively, we can obtain the first-order conditions as follows:

(7.1) $$\frac{d\tilde{\pi}_C}{d\tilde{c}} = \overbrace{\tilde{Q}}^{\text{marginal benefit effect}(+)} + \overbrace{\tilde{c}\frac{\partial\tilde{Q}}{\partial\tilde{c}}}^{\text{marginal cost effect}(-)} = \left(\frac{2-2\tilde{c}-\tilde{s}+\tilde{\theta}_1+\tilde{\theta}_2}{3}\right)+\tilde{c}\left(-\frac{2}{3}\right)=0$$

(7.2) $$\frac{d\tilde{\pi}_S}{d\tilde{s}} = \overbrace{\tilde{q}_1}^{\text{marginal benefit effect}(+)} + \overbrace{\tilde{s}\frac{\partial\tilde{q}_1}{\partial\tilde{s}}}^{\text{marginal cost effect}(-)} = \left(\frac{1-\tilde{c}-2\tilde{s}+2\tilde{\theta}_1-\tilde{\theta}_2}{3}\right)+\tilde{s}\left(-\frac{2}{3}\right)=0$$

The optimal input prices $\tilde{c}$ and $\tilde{s}$ are determined by the balance of the *marginal benefit effect* and the *marginal cost effect* denoted by the first and second terms on the right-hand side of the first-order conditions in (7.1) and (7.2) in that order. The marginal benefit effect equals the sale of the input and is positive as a rise in the input price increases the profit directly. On the other hand, a higher input price reduces the derived demand for the input, leading to the negative marginal cost effect. Moreover, Eq. (7.1) shows that the presence of the supplementary input reduces supplier *C*'s marginal benefit in choosing an optimal input price. Hence, given the incentive parameters, the optimal $\tilde{c}$ becomes lower than that in the absence of input *S*.

Solving (7) gives the optimal input prices as:

(8) $$\tilde{c} = \frac{7}{15}+\frac{2}{15}\tilde{\theta}_1+\frac{1}{3}\tilde{\theta}_2, \tilde{s} = \frac{2}{15}+\frac{7}{15}\tilde{\theta}_1-\frac{1}{3}\tilde{\theta}_2$$

From (8), we can find that both increases in incentive parameters of firms 1 and 2 cause a rise in the uniform price of input *C*. However, the impact of a change in $\tilde{\theta}_2$ on $\tilde{c}$ is more substantial than that of $\tilde{\theta}_1$. On the other hand, a higher $\tilde{\theta}_1$ increases the input price for input *S*, while the opposite influence arises with a higher $\tilde{\theta}_2$ but in a lower magnitude.

We now move to the first stage, where the downstream firms' owners determine whether to delegate or not. The equilibrium results under the case both firms choose not to delegate (NN) can be derived by substituting $\tilde{\theta}_1 = \tilde{\theta}_2 = 0$ into (3), (4), (6), and (8). Calculations lead to the following:

$$\tilde{c}^{NN} = \frac{7}{15}, \tilde{s}^{NN} = \frac{2}{15} \tag{9.1}$$

$$\tilde{q}_1^{NN} = \frac{4}{45}, \tilde{q}_2^{NN} = \frac{2}{9} \tag{9.2}$$

$$\tilde{\pi}_C^{NN} = \frac{98}{675}, \tilde{\pi}_S^{NN} = \frac{8}{675} \tag{9.3}$$

$$\tilde{\pi}_1^{NN} = \frac{16}{2025}, \tilde{\pi}_2^{NN} = \frac{4}{81} \tag{9.4}$$

Next, if only firm 2 (firm 1) chooses to delegate, i.e., case ND (DN), we substitute $\tilde{\theta}_1 = 0 (\tilde{\theta}_2 = 0)$ and (6) and (8) into (4.1), then maximizing $\tilde{\pi}_2 (\tilde{\pi}_1)$ with respect to $\tilde{\theta}_2$ ($\tilde{\theta}_1$). The results are obtained in Table 1 hereunder.

TABLE 1

THE EQUILIBRIUM RESULTS OF ASYMMETRIC DELEGATION UNDER UNIFORM INPUT PRICING

$\tilde{\theta}_1^{ND} = 0, \tilde{\theta}_2^{ND} = -\frac{1}{20} < 0$	$\tilde{\theta}_1^{DN} = -\frac{17}{217} < 0, \tilde{\theta}_2^{DN} = 0$
$\tilde{c}^{ND} = \frac{9}{20}, \tilde{s}^{ND} = \frac{3}{20}$	$\tilde{c}^{DN} = \frac{99}{217}, \tilde{s}^{DN} = \frac{3}{31}$
$\tilde{q}_1^{ND} = \frac{1}{10}, \tilde{q}_2^{ND} = \frac{1}{5}$	$\tilde{q}_1^{DN} = \frac{2}{31}, \tilde{q}_2^{DN} = \frac{52}{217}$
$\tilde{\pi}_C^{ND} = \frac{27}{200}, \tilde{\pi}_S^{ND} = \frac{3}{200}$	$\tilde{\pi}_C^{DN} = \frac{6534}{47089}, \tilde{\pi}_S^{DN} = \frac{6}{961}$
$\tilde{\pi}_1^{ND} = \frac{1}{100}, \tilde{\pi}_2^{ND} = \frac{1}{20}$	$\tilde{\pi}_1^{DN} = \frac{2}{217}, \tilde{\pi}_2^{DN} = \frac{2704}{47089}$

Lastly, let us consider the subgame in which both downstream owners choose to delegate (DD). The first-order conditions are derived implicitly as follows:

$$\frac{d\tilde{\pi}_i}{d\tilde{\theta}_i} = \overbrace{\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_i}\frac{d\tilde{q}_i}{d\tilde{\theta}_i}}^{\text{output effect (?)}} + \overbrace{\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_j}\frac{\partial\tilde{q}_j}{\partial\tilde{\theta}_i}}^{\text{strategic effect (+)}} + \overbrace{\frac{\partial\tilde{\pi}_i}{\partial\tilde{k}_i}\frac{\partial\tilde{k}_i}{\partial\tilde{\theta}_i}}^{\text{vertical externality effect (−)}} + \overbrace{\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_j}\frac{\partial\tilde{q}_j}{\partial\tilde{c}}\frac{\partial\tilde{c}}{\partial\tilde{\theta}_i}}^{\text{spillover effect (+)}} + \overbrace{\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_j}\frac{\partial\tilde{q}_j}{\partial\tilde{s}}\frac{\partial\tilde{s}}{\partial\tilde{\theta}_i}}^{\text{supplementary input effect (−)}} = 0;\ i,j=1,2; i\neq j \tag{10}$$

where $\frac{d\tilde{q}_i}{d\tilde{\theta}_i} = \frac{\partial\tilde{q}_i}{\partial\tilde{\theta}_i} + \frac{\partial\tilde{q}_i}{\partial\tilde{c}}\frac{\partial\tilde{c}}{\partial\tilde{\theta}_i} + \frac{\partial\tilde{q}_i}{\partial\tilde{s}}\frac{\partial\tilde{s}}{\partial\tilde{\theta}_i}$.

It is found from (10) that firm *i*'s (i = 1, 2) optimal incentive parameter is manipulated by the balance of the *output effect*, the *strategic effect*, the *vertical externality effect*, the *spillover effect*, and the *supplementary input effect*, as in the right-hand side of the equation. The output effect indicates that a rise in firm *i*'s incentive parameter induces its manager to compete more aggressively by increasing its output. The higher output quantity then improves (lowers) the firm's profit if the incentive parameter is negative (positive). Thus, the output effect is ambiguous.[11] Next, the strategic effect shows that an increase in firm *i*'s incentive parameter reduces its rival's output by strategic substitutes. Then a lower quantity supplied by the opposition raises the price of the final product, leading to higher profit for firm *i*. The strategic effect is, therefore, positive.[12]

The vertical externality effect exhibits that if firm *i*'s manager competes more severely in the output market, the upstream firm(s) will charge a higher price such that its marginal cost increases. And the rise in the marginal cost reduces firm *i*'s output quantity and its profit. Thus, this effect is negative.[13] On the contrary, the spillover effect is positive as a rise in $\tilde{\theta}_i$ increases input *C*'s uniform price such that firm *j*'s marginal cost ($i, j = 1, 2; i \neq j$) also grows, leading to less output produced by firm *j*. The price, therefore, increases, and firm *i*'s profit is improved.[14] Lastly, the supplementary input effect encourages firm 1 (firm 2) to lower its incentive parameter to reduce (raise) supplier *S*'s input price, leading to lower output produced by its rival. The price then in-

11 By (6) and (8), we can calculate that $\frac{d\tilde{q}_1}{d\tilde{\theta}_1} = \frac{14}{45} > 0$ and $\frac{d\tilde{q}_2}{d\tilde{\theta}_2} = \frac{4}{9} > 0$. Moreover, it is derived from (5) that $\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_i} = -\tilde{\theta}_i > (<) 0 \text{ if } \tilde{\theta}_i < (>) 0, i = 1,2$. Hence, the output effect is ambiguous.

12 From (6), we can find that $\frac{\partial\tilde{q}_j}{\partial\tilde{\theta}_i} = -\frac{1}{3} < 0, i, j = 1,2, i \neq j$. Also, it is shown by (4.1) that $\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_j} = -\tilde{q}_i < 0$. Accordingly, the strategic effect is positive.

13 By (8), we can derive that $\frac{\partial\tilde{k}_1}{\partial\tilde{\theta}_1} = \frac{\partial\tilde{c}}{\partial\tilde{\theta}_1} + \frac{\partial\tilde{s}}{\partial\tilde{\theta}_1} = \frac{3}{5} > 0$ and $\frac{\partial\tilde{k}_1}{\partial\tilde{\theta}_2} = \frac{\partial\tilde{c}}{\partial\tilde{\theta}_2} = \frac{1}{3} > 0$. It is also found from (4.1) that $\frac{\partial\tilde{\pi}_i}{\partial\tilde{k}_i} = -\tilde{q}_i < 0$. The vertical externality effect is, therefore, negative.

14 Computations from (4.1), (6), and (8) show that $\frac{\partial\tilde{\pi}_i}{\partial\tilde{q}_j} = -\tilde{q}_i < 0$, $\frac{\partial\tilde{q}_j}{\partial\tilde{c}} < 0$, and $\frac{\partial\tilde{c}}{\partial\tilde{\theta}_i} > 0$, $i, j = 1,2, i \neq j$. Hence, the spillover effect is positive.

creases, and the firm in question is better off. Accordingly, the supplementary input effect is negative.[15]

Solving (10) for the optimal incentive parameters gives:

(11.1) $$\tilde{\theta}_1^{DD} = -\frac{17}{191} < 0, \tilde{\theta}_2^{DD} = -\frac{52}{955} < 0$$

From (11.1), it is found that both managers behave less aggressively in the output market. The presence of the supplementary input effect helps the negative effects become more substantial than the positive effects such that their balance is at a negative incentive parameter.

Substituting (11.1) into (3), (4), (6), and (8), we can obtain:

(11.2) $$\tilde{c}^{DD} = \frac{417}{955}, \tilde{s}^{DD} = \frac{21}{191}$$

(11.3) $$\tilde{q}_1^{DD} = \frac{14}{191}, \tilde{q}_2^{DD} = \frac{208}{955}$$

(11.4) $$\tilde{\pi}_C^{DD} = \frac{115926}{912025}, \tilde{\pi}_S^{DD} = \frac{294}{36481}$$

(11.5) $$\tilde{\pi}_1^{DD} = \frac{434}{36481}, \tilde{\pi}_2^{DD} = \frac{10816}{182405}$$

By (9.4), (11.5), and Table 1, we can get the following:

(12.1) $$\tilde{\pi}_1^{DN} - \tilde{\pi}_1^{NN} = \frac{578}{439425} > 0; \tilde{\pi}_1^{DD} - \tilde{\pi}_1^{ND} = \frac{6919}{3648100} > 0$$

(12.2) $$\tilde{\pi}_2^{ND} - \tilde{\pi}_2^{NN} = \frac{1}{1620} > 0; \tilde{\pi}_2^{DD} - \tilde{\pi}_2^{DN} = \frac{16091504}{8589269045} > 0$$

It is shown from (12) that to delegate is the dominant strategy for both firm 1 and firm 2. As a result, they both delegate in equilibrium.[16] Next, I examine whether delegation is mutually profitable or unprofitable for the downstream firms by the following:

(13) $$\tilde{\pi}_1^{DD} - \tilde{\pi}_1^{NN} = \frac{295154}{73874025} > 0; \tilde{\pi}_2^{DD} - \tilde{\pi}_2^{NN} = \frac{146476}{14774805} > 0$$

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15 It is found from (4.1) that $\frac{\partial \tilde{\pi}_i}{\partial \tilde{q}_j} = -\tilde{q}_i < 0, i, j = 1, 2, i \neq j$. Next, we can calculate from (6) and (8) that $\frac{\partial \tilde{q}_2}{\partial \tilde{s}} \frac{\partial \tilde{s}}{\partial \tilde{\theta}_1} = \frac{1}{3} \times \frac{7}{15} = \frac{7}{45} > 0$ and $\frac{\partial \tilde{q}_1}{\partial \tilde{s}} \frac{\partial \tilde{s}}{\partial \tilde{\theta}_2} = \left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right) = \frac{2}{9} > 0$. Thus, the supplementary input effect is negative.

16 Since not to delegate is a particular case of to delegate, where the owner sets the incentive parameter to be zero, we can conclude that both firms delegate in equilibrium if both parameters are non-zero under subgame DD.

The inequalities in (13) indicate that both downstream firms are better off in equilibrium compared to the case that no firm delegates under the uniform input pricing regime. Moreover, Appendix A shows this result also occurs under third-degree input price discrimination. Accordingly, I establish the following proposition:

Proposition 1. *Both downstream firms delegate in equilibrium, but contrary to traditional analysis, delegation is mutually profitable, no matter whether the core input supplier charges uniform or discriminatory prices.*

Proposition 1 is sharply different from Liao (2010), where delegation is still mutually unprofitable under uniform input pricing. Recall the effects in (10). It should be noted that there is no supplementary input effect in the model of Liao (2010). In that paper, the downstream firms behave less forcefully post-delegation under discriminatory input pricing because of the vertical externality effect, leading to higher profits. When the upstream firm charges uniform input prices, there exists the extra spillover effect that helps outweigh the vertical externality effect such that the firms compete more severely post-delegation, leading to lower profits. Notably, the current paper creates the supplementary input effect that induces both firms to reduce the intensity of competition post-delegation. Hence, this effect amplifies the result of Liao (2010) under discriminatory input pricing while making the reverse result occur under uniform input pricing.

3. Uniform Vs Discriminatory Input Pricing

This section investigates which pricing strategy is superior from the viewpoint of the core input supplier. In addition, a welfare comparison between uniform and discriminatory input pricing will also be taken into consideration. Let us start by computing supplier C's profit difference between the two input pricing regimes. By subtracting $\hat{\pi}_C^{DD}$ in (A.4.4) from $\tilde{\pi}_C^{DD}$ in (11.4), we can obtain:

$$\tilde{\pi}_C^{DD} - \hat{\pi}_C^{DD} = \frac{645097}{17875690} > 0 \qquad (14)$$

The inequality in (14) indicates that the core input supplier prefers uniform pricing to third-degree price discrimination. The rationale behind this result is that the spillover effect emerges under uniform input pricing such that the downstream firms are less willing to strategically lower the input price.[17] As a result, the core input supplier is better off under uniform input pricing.

I mark the above result by the following proposition:

Proposition 2. *The core input supplier prefers uniform input pricing to third-degree price discrimination.*

Although Proposition 2 is consistent with the finding by Liao (2010, p. 268), this result is significantly different from that of Brito *et al*. (2019), in which input price discrimination benefits the upstream monopolist if there is downstream asymmetry under a non-delegation framework. While their paper incorporates quality asymmetry, the current study considers the downstream asymmetry as the difference in the number of inputs employed by the downstream firms. Constructing a managerial delegation game, I find that uniform input pricing is superior from the viewpoint of the core input supplier.

In what follows, I examine the welfare ranking between two pricing strategies. By substituting (11) and (A.4) into the utility function in (1), we can obtain the welfare levels (W) under the uniform and discriminatory input pricing regimes, respectively. Then, comparing them gives:

$$\tilde{W}^{DD} - \hat{W}^{DD} = \frac{226848}{912025} - \frac{2091}{9800} = \frac{2528529}{71502760} > 0 \quad (15)$$

Even though the downstream firms always compete less aggressively than profit-maximizers, Footnote 17 shows that the competition intensity is stronger under uniform than discriminatory input pricing due to the spillover effect. This implies that the downstream firms produce more output under the former regime.[18] Thus, consumer surplus and welfare are higher under uniform input pricing. Based on this result, I establish the following:

Proposition 3. *Uniform input pricing is more socially desirable than third-degree input price discrimination.*

Proposition 3 supports the traditional result by DeGraba (1990) and Valletti (2003), in which price discrimination provides more instruments for the upstream monopolist to capture profits from the downstream firms, causing lower total output produced in the downstream market and lower welfare.

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17 We can observe this by comparing the incentive parameters between the two regimes. From (11.1) and (A.4.1), the comparison shows that $\tilde{\theta}_i > \hat{\theta}_i, i = 1,2$ as $\tilde{\theta}_1 - \hat{\theta}_1 = \frac{335}{2764} > 0$ and $\tilde{\theta}_2 - \hat{\theta}_2 = \frac{139}{955} > 0$.

18 Calculating from (11.3) and (A.4.3), we can find that $\tilde{Q}^{DD} = \frac{278}{955} > \hat{Q}^{DD} = \frac{17}{70}$.

4. Downstream Vertically Differentiated Products

In this section, I discuss the case where the downstream firms produce vertically (quality) differentiated products. By assuming $a_1 = \alpha$ and $a_2 = 1$ in (1) and (2), we can use α to measure the quality asymmetry of the downstream products. The restriction $\alpha \in \left(\frac{5}{7}, 2\right)$ is assumed to be held throughout the analysis to guarantee positive output quantities. If $\alpha > (<) 1$, the multi-input firm produces higher (lower) quality products than the single-input firm.[19] If $\alpha = 1$, the downstream products are homogeneous, so the analysis degenerates to the primary model. To save space, I move all the proofs to Appendix B.

Appendix B shows that the results in Propositions 1 – 3 are robust when the downstream firms produce vertically differentiated products. These results provide several interesting comparisons with the existing literature. First, when considering the downstream market only, Colombo (2022) indicates that the result of VFJS may not hold, as the high-quality firm will benefit from delegation if the quality asymmetry is high enough. However, Appendix B demonstrates that both downstream firms in the current model benefit from delegation, regardless of quality differential. The reason is based on the vertical externality and the supplementary effects that influence the downstream firms to reduce their aggressiveness in the output market. This leads to less intense competition, which benefits both firms.

Next, considering quality asymmetry in a non-delegation model, Brito *et al.* (2019) show that an upstream monopolist prefers third-degree price discrimination to uniform pricing. On the contrary, I prove that the core input supplier prefers uniform pricing to third-degree price discrimination by introducing quality and technological asymmetry in the downstream delegation game. The contrast may come from the model settings. In addition to the difference mentioned above, Brito *et al.* (2019) adopt the model of vertical differentiation by Choi and Shin (1992) and Motta (1993), while I measure the quality of the product by the consumers' willingness to pay as in Häckner (2000). Moreover, Brito *et al.* (2019) assume Bertrand competition in the downstream market while this paper takes into consideration the Cournot case.

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19 The case of $\alpha > 1$ happens when firm 1 uses an advanced technology that requires extra input to enhance the quality of the product. On the contrary, $\alpha \leq 1$ occurs when firm 1 uses an outdated technology that requires extra input to handle the disadvantage, such as labor, security, IT support, and maintenance services. Please refer to the website in Footnote 3 for more information.

5. Concluding Remarks

This paper has studied a delegation game between one multi-input firm and one single-input firm competing in quantity. Assuming both firms employ standard input from the core input supplier, and the multi-input firm requires extra input from the supplementary input supplier, we have also discussed the profit and welfare implications of two input pricing strategies: uniform pricing and third-degree price discrimination.

Three results are obtained from the analysis. First, regardless of the upstream pricing strategies, both downstream firms delegate in equilibrium, but contrary to traditional conclusion, delegation is mutually profitable. Second, the core input supplier prefers uniform pricing to third-degree price discrimination. Lastly, uniform pricing is more socially desirable than discriminatory pricing.

These results are specific to the assumption of linear wholesale pricing to focus on comparing uniform pricing and third-degree price discrimination. However, it would also be interesting for the future study to allow two-part tariffs. Another issue that stays outside this paper's scope but deserves to be considered is the strategic vertical integration/separation of the firms by following Bonanno and Vickers (1988).

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Appendix A

This appendix discusses the regime where supplier C can price discriminate. Maximizing V_i in (4.2), we can obtain the optimal output quantities in stage 3 as:

$$\text{(A.1)} \quad \hat{q}_1 = \frac{1-2\hat{c}_1+\hat{c}_2-2\hat{s}+2\hat{\theta}_1-\hat{\theta}_2}{3}, \hat{q}_2 = \frac{1+\hat{c}_1-2\hat{c}_2+\hat{s}-\hat{\theta}_1+2\hat{\theta}_2}{3}$$

where the accent ^ denotes variables under third-degree input price discrimination.

In stage 2, I substitute (A.1) into (3), then solve for the profit-maximizing input prices as follows:

$$\text{(A.2)} \quad \hat{c}_1 = \frac{5+4\hat{\theta}_1+\hat{\theta}_2}{12}, \hat{c}_2 = \frac{1+\hat{\theta}_2}{2}, \hat{s} = \frac{1+2\hat{\theta}_1-\hat{\theta}_2}{6}$$

In the first stage, by substituting $\hat{\theta}_1 = \hat{\theta}_2 = 0$ into (3), (4), (A.1), and (A.2), the equilibrium results when both firms choose not to delegate (NN) are obtained as:

$$\text{(A.3.1)} \quad \hat{c}_1^{NN} = \frac{5}{12}, \hat{c}_2^{NN} = \frac{1}{2}, \hat{s}^{NN} = \frac{1}{6}$$

$$\text{(A.3.2)} \quad \hat{q}_1^{NN} = \frac{1}{9}, \hat{q}_2^{NN} = \frac{7}{36}$$

$$\text{(A.3.3)} \quad \hat{\pi}_C^{NN} = \frac{31}{216}, \hat{\pi}_S^{NN} = \frac{1}{54}$$

$$\text{(A.3.4)} \quad \hat{\pi}_1^{NN} = \frac{1}{81}, \hat{\pi}_2^{NN} = \frac{49}{1296}$$

By substituting $\hat{\theta}_1 = 0$ ($\hat{\theta}_2 = 0$) and (A.2) into firm 2's (1's) profit function and maximizing it with respect to $\hat{\theta}_2$ ($\hat{\theta}_1$), we can acquire the equilibrium results in the case where only firm 2 (1) delegates, i.e., case ND (DN), as in the following table:

TABLE A.1
THE EQUILIBRIUM RESULTS OF ASYMMETRIC DELEGATION UNDER INPUT PRICE DISCRIMINATION

$\hat{\theta}_1^{ND}=0, \hat{\theta}_2^{ND}=-\frac{49}{275}<0$	$\hat{\theta}_1^{DN}=-\frac{5}{28}<0, \hat{\theta}_2^{DN}=0$
$\hat{c}_1^{ND}=\frac{221}{550}, \hat{c}_2^{ND}=\frac{113}{275}, \hat{s}^{ND}=\frac{54}{275}$	$\hat{c}_1^{DN}=\frac{5}{14}, \hat{c}_2^{DN}=\frac{1}{2}, \hat{s}^{DN}=\frac{3}{28}$
$\hat{q}_1^{ND}=\frac{36}{275}, \hat{q}_2^{ND}=\frac{7}{50}$	$\hat{q}_1^{DN}=\frac{1}{14}, \hat{q}_2^{DN}=\frac{3}{14}$
$\hat{\pi}_C^{ND}=\frac{16657}{151250}, \hat{\pi}_S^{ND}=\frac{1944}{75625}$	$\hat{\pi}_C^{DN}=\frac{13}{98}, \hat{\pi}_S^{DN}=\frac{3}{392}$
$\hat{\pi}_1^{ND}=\frac{1296}{75625}, \hat{\pi}_2^{ND}=\frac{49}{1100}$	$\hat{\pi}_1^{DN}=\frac{1}{56}, \hat{\pi}_2^{DN}=\frac{9}{196}$

By substituting (A.2) into the firms' profit functions and solving for the optimal incentive parameters when both firms choose to delegate (case DD), I show that:

(A.4.1) $$\hat{\theta}_1^{DD}=-\frac{3}{14}<0, \hat{\theta}_2^{DD}=-\frac{1}{5}<0$$

Substituting (A.4.1) into (3), (4), (A.1), and (A.2) gives:

(A.4.2) $$\hat{c}_1^{DD}=\frac{23}{70}, \hat{c}_2^{DD}=\frac{2}{5}, \hat{s}^{DD}=\frac{9}{70}$$

(A.4.3) $$\hat{q}_1^{DD}=\frac{3}{35}, \hat{q}_2^{DD}=\frac{11}{70}$$

(A.4.4) $$\hat{\pi}_C^{DD}=\frac{223}{2450}, \hat{\pi}_S^{DD}=\frac{27}{2450}$$

(A.4.5) $$\hat{\pi}_1^{DD}=\frac{9}{350}, \hat{\pi}_2^{DD}=\frac{11}{196}$$

By (A.3.4), Table A.1, and (A.4.5), we can derive that $\hat{\pi}_1^{DN}-\hat{\pi}_1^{NN}=\frac{25}{4536}>0$, $\hat{\pi}_1^{DD}-\hat{\pi}_1^{ND}=\frac{9081}{1058750}$, $\hat{\pi}_2^{ND}-\hat{\pi}_2^{NN}=\frac{2401}{356400}>0$, and $\hat{\pi}_2^{DD}-\hat{\pi}_2^{DN}=\frac{1}{98}>0$. It follows that to delegate the output decision is the dominant strategy for both firms, resulting in DD being a unique equilibrium.

Moreover, comparisons also give $\hat{\pi}_1^{DD}-\hat{\pi}_1^{NN}=\frac{379}{28350}>0$ and $\hat{\pi}_2^{DD}-\hat{\pi}_2^{NN}=\frac{1163}{63504}>0$. As a result, delegation is mutually profitable for both downstream firms under input price discrimination.

Appendix B

This appendix provides the analytical results of the case where the downstream firms produce quality differentiated products. Given the demand system $p_1 = \alpha - Q$ and $p_2 = 1 - Q$ with $\alpha \in \left(\frac{5}{7}, 2\right)$, I derive the results under uniform (discriminatory) input pricing as in Table B.1 (Table B.2) through the same calculation process as in the primary model.

TABLE B.1

RESULTS OF FOUR SUBGAMES UNDER UNIFORM INPUT PRICING

	NN	DD	DN	ND
$\left(\tilde{\theta}_1, \tilde{\theta}_2\right)$	$(0,0)$	$\left(\frac{17(2-3\alpha)}{191}, \frac{35\alpha-87}{955}\right)$	$\left(\frac{85}{434} - \frac{17\alpha}{62}, 0\right)$	$\left(0, \frac{\alpha-2}{20}\right)$
$\left(\tilde{c}, \tilde{s}\right)$	$\left(\frac{5+2\alpha}{15}, \frac{7\alpha-5}{15}\right)$	$\left(\frac{3(104+35\alpha)}{955}, \frac{21(3\alpha-2)}{191}\right)$	$\left(\frac{78}{217} + \frac{3\alpha}{31}, \frac{21\alpha-15}{62}\right)$	$\left(\frac{3(\alpha+2)}{20}, \frac{3(3\alpha-2)}{20}\right)$
$\left(\tilde{q}_1, \tilde{q}_2\right)$	$\left(\frac{2(7\alpha-5)}{45}, \frac{2(2-\alpha)}{9}\right)$	$\left(\frac{14(3\alpha-2)}{191}, \frac{4(87-35\alpha)}{955}\right)$	$\left(\frac{7\alpha-5}{31}, \frac{87}{217} - \frac{5\alpha}{31}\right)$	$\left(\frac{3\alpha-2}{10}, \frac{2-\alpha}{5}\right)$
$\left(\tilde{\pi}_C, \tilde{\pi}_S\right)$	$\left(\frac{2(5+2\alpha)^2}{675}, \frac{2(7\alpha-5)^2}{675}\right)$	$\left(\frac{6(104+35\alpha)^2}{912025}, \frac{294(3\alpha-2)^2}{36481}\right)$	$\left(\frac{6(26+7\alpha)^2}{47089}, \frac{3(7\alpha-5)^2}{1922}\right)$	$\left(\frac{3(2+\alpha)^2}{200}, \frac{3(3\alpha-2)^2}{200}\right)$
$\left(\tilde{\pi}_1, \tilde{\pi}_2\right)$	$\left(\frac{4(7\alpha-5)^2}{2025}, \frac{4(2-\alpha)^2}{81}\right)$	$\left(\frac{434(3\alpha-2)^2}{36481}, \frac{4(87-35\alpha)^2}{182405}\right)$	$\left(\frac{(7\alpha-5)^2}{434}, \frac{(35\alpha-87)^2}{47089}\right)$	$\left(\frac{(3\alpha-2)^2}{100}, \frac{(2-\alpha)^2}{20}\right)$

TABLE B.2

RESULTS OF FOUR SUBGAMES UNDER DISCRIMINATORY INPUT PRICING

	NN	DD	DN	ND
$\left(\hat{\theta}_1, \hat{\theta}_2\right)$	$(0,0)$	$\left(\frac{11-29\alpha}{84}, \frac{\alpha-4}{15}\right)$	$\left(\frac{5(1-2\alpha)}{28}, 0\right)$	$\left(0, \frac{28\alpha-77}{275}\right)$
$\left(\hat{c}_1, \hat{c}_2, \hat{s}\right)$	$\left(\frac{1+4\alpha}{12}, \frac{1}{2}, \frac{2\alpha-1}{6}\right)$	$\left(\frac{47\alpha+22}{210}, \frac{\alpha+11}{30}, \frac{29\alpha-11}{140}\right)$	$\left(\frac{2+3\alpha}{14}, \frac{1}{2}, \frac{3(2\alpha-1)}{28}\right)$	$\left(\frac{3}{50} + \frac{94\alpha}{275}, \frac{14\alpha+99}{275}, \frac{87\alpha-33}{275}\right)$
$\left(\hat{q}_1, \hat{q}_2\right)$	$\left(\frac{2\alpha-1}{9}, \frac{11-4\alpha}{36}\right)$	$\left(\frac{29\alpha-11}{210}, \frac{11(4-\alpha)}{210}\right)$	$\left(\frac{2\alpha-1}{14}, \frac{4-\alpha}{14}\right)$	$\left(\frac{58\alpha-22}{275}, \frac{11-4\alpha}{50}\right)$
$\left(\hat{\pi}_C, \hat{\pi}_S\right)$	$\left(\frac{2}{27}\alpha^2 - \frac{2}{27}\alpha + \frac{31}{216}, \frac{(2\alpha-1)^2}{54}\right)$	$\left(\frac{643\alpha^2-209\alpha+1573}{22050}, \frac{(29\alpha-11)^2}{29400}\right)$	$\left(\frac{3\alpha^2-3\alpha+13}{98}, \frac{3(2\alpha-1)^2}{392}\right)$	$\left(\frac{5144\alpha^2}{75625} - \frac{222\alpha}{6875} + \frac{93}{1250}, \frac{6(29\alpha-11)^2}{75625}\right)$
$\left(\hat{\pi}_1, \hat{\pi}_2\right)$	$\left(\frac{(2\alpha-1)^2}{81}, \frac{(11-4\alpha)^2}{1296}\right)$	$\left(\frac{(29\alpha-11)^2}{12600}, \frac{11(4-\alpha)^2}{1764}\right)$	$\left(\frac{(2\alpha-1)^2}{56}, \frac{(4-\alpha)^2}{196}\right)$	$\left(\frac{4(29\alpha-11)^2}{75625}, \frac{(11-4\alpha)^2}{1100}\right)$

Calculations from Table B.1 show that:

(B.1.1) $$\tilde{\pi}_1^{DN} - \tilde{\pi}_1^{NN} = \frac{289(7\alpha - 5)^2}{878850} > 0, \tilde{\pi}_1^{DD} - \tilde{\pi}_1^{ND} = \frac{6919(3\alpha - 2)^2}{3648100} > 0$$

(B.1.2) $$\tilde{\pi}_2^{ND} - \tilde{\pi}_2^{NN} = \frac{(2-\alpha)^2}{1620} > 0, \tilde{\pi}_2^{DD} - \tilde{\pi}_2^{DN} = \frac{5951(87 - 35\alpha)^2}{8589269045} > 0$$

(B.1.3) $$\tilde{\pi}_1^{DD} - \tilde{\pi}_1^{NN} = \frac{759374\left(\alpha - \frac{11840}{54241} - \frac{8595\sqrt{434}}{379687}\right)\left(\alpha - \frac{11840}{54241} + \frac{8595\sqrt{434}}{379687}\right)}{73874025} > 0$$

(B.1.4) $$\tilde{\pi}_2^{DD} - \tilde{\pi}_2^{NN} = -\frac{66544\left(\alpha - \frac{23633}{16636} + \frac{29223\sqrt{5}}{83180}\right)\left(\alpha - \frac{23633}{16636} - \frac{29223\sqrt{5}}{83180}\right)}{2954961} > 0$$

since $\alpha \in \left(\frac{5}{7}, 2\right)$.

It is found from (B.1.1) and (B.1.2) that to delegate is the dominant strategy for both firms, and DD is a unique equilibrium under uniform input pricing. Moreover, (B.1.3) and (B.1.4) indicate that the firms are better off post-delegation. This is because both firms behave less aggressively than profit-maximizers in equilibrium as $\tilde{\theta}_i^{DD}(i = 1,2) < 0$ shown in Table B.1.

Similarly, I compute from Table B.2 that:

(B.2.1) $$\hat{\pi}_1^{DN} - \hat{\pi}_1^{NN} = \frac{25(2\alpha - 1)^2}{4536} > 0, \hat{\pi}_1^{DD} - \hat{\pi}_1^{ND} = \frac{1009(29\alpha - 11)^2}{38115000} > 0$$

(B.2.2) $$\hat{\pi}_2^{ND} - \hat{\pi}_2^{NN} = \frac{49(11 - 4\alpha)^2}{356400} > 0, \hat{\pi}_2^{DD} - \hat{\pi}_2^{DN} = \frac{(4-\alpha)^2}{882} > 0$$

(B.2.3) $$\hat{\pi}_1^{DD} - \hat{\pi}_1^{NN} = \frac{1969}{113400}\left(\alpha - \frac{71}{1969} - \frac{210\sqrt{14}}{1969}\right)\left(\alpha - \frac{71}{1969} + \frac{210\sqrt{14}}{1969}\right) > 0$$

(B.2.4) $$\hat{\pi}_2^{DD} - \hat{\pi}_2^{NN} = -\frac{97}{15876}\left(\alpha - \frac{143}{97} + \frac{105\sqrt{11}}{194}\right)\left(\alpha - \frac{143}{97} - \frac{105\sqrt{11}}{194}\right) > 0$$

since $\alpha \in \left(\frac{5}{7}, 2\right)$.

The inequalities in (B.2) demonstrate that both firms delegate in equilibrium, but they benefit from delegation under discriminatory input pricing. The reason is based on the less intense downstream competition, as shown by $\hat{\theta}_i^{DD}(i = 1,2) < 0$ in Table B.2.

Next, comparing $\tilde{\pi}_C^{DD}$ and $\hat{\pi}_C^{DD}$ from Tables B.1 and B.2 yields:

(B.3) $$\tilde{\pi}_C^{DD} - \hat{\pi}_C^{DD} = \frac{16974583\left(\alpha - \frac{46150289}{33949166} + \frac{63\sqrt{534117627861}}{33949166}\right)\left(\alpha - \frac{46150289}{33949166} - \frac{63\sqrt{534117627861}}{33949166}\right)}{804406050} > 0$$

since $\alpha \in \left(\frac{5}{7}, 2\right)$.

It is shown by (B.3) that supplier C prefers uniform input pricing to third-degree price discrimination.

Lastly, I derive the equilibrium welfare levels under the two regimes by substituting the optimal output quantities in Tables B.1 and B.2 into the utility function in (1). Comparing them gives:

$$\text{(B.4)} \qquad \tilde{W}^{DD} - \hat{W}^{DD} = \frac{11099086}{134067675}\left(\alpha - \frac{51194251}{44396344}\right)^2 + \frac{148380153}{4439634400} > 0$$

From (B.4), we can find that uniform input pricing is welfare superior to third-degree input price discrimination.